Int. J. Heat Mass Transfer. Vol. 27, No. 6, pp. 857–862, 1984
Printed in Great Britain

0017-9310/84 $3.00 + 0.00
Pergamon Press Ltd.

HEAT AND MASS TRANSFER IN VISCOUS SWIRLED JETS

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(*Received* 9 *September* 1982)

Abstract—Within the framework of the Loitsyansky theory for the problem of the development of submerged swirled jets, the distribution of excess temperatures in swirled jets has been obtained by the method of asymptotic expansions. The second and third terms of the excess temperature expansion have been found taking account of the effect of jet swirling intensity over a wide range of Prandtl numbers ($Pr \neq 1$). The results of experimental investigations of the velocity, pressure and concentration fields of a gas admixture in turbulent swirled jets of air are given which are compared with the prediction.

NOMENCLATURE

a	thermal diffusivity
A	eddy viscosity coefficient
c	gas admixture concentration
c_0	initial concentration
c_p	specific heat at constant pressure
M_0	initial mass rate of flow
P	pressure
Q_0	heat flux
Sc	Schmidt number
ΔT	excess temperature as against that of the surrounding medium
U	value of the mean flow speed
u, v, w	axial and radial velocity components and rate of swirling
x, r	longitudinal and transverse coordinates.

Greek symbols

α	$(3K_0/16\pi A)^{1/2}$
β	$M_0/2\pi A$
γ	$3\alpha L_0/16\pi A v_t^{1/2}$
δ	characteristic constants of swirled jets [6]
v_t	coefficient of kinematic eddy viscosity.

INTRODUCTION

A LARGE number of works have been reported which deal with the study of the process of formation and development of laminar and turbulent swirled jets in an infinite space submerged in a medium with the same physical properties as the jet. A summary of the main works can be found in refs. [1–4].

Unlike straight jets, the swirled jets are characterized by the presence of the centrifugal effect which produces transverse and longitudinal pressure gradients. This explains their wide use in different technological processes and in engineering. For example, in furnace engineering, the swirled jets are used as a source of acceleration and stabilization of burning. They have a larger divergence angle and accordingly a smaller range, as compared with straight jets, and a higher ejection ability which makes for better mixing of a fuel with air, while the recirculation region originating at the root of an intensely swirled jet favours the process of burning stabilization.

The statement and the first analytical solution of the problem of swirled jet development in an infinite space are due to Loitsyansky [5]. The problem was solved on the basis of the laminar boundary-layer equations by the method of asymptotic expansions. Loitsyansky has found the first and second terms of velocity and pressure components in their final form. The third and fourth terms of the velocity expansion were determined in ref. [6] and of the pressure expansion in ref. [3]. The solution makes it possible to allow for the effect of swirling intensity on the velocity and pressure distribution in a jet and to evaluate the extension of the recirculation region. Heat transfer in a swirled jet at $Pr = 1$ has been considered elsewhere [8].

In this paper, the distribution of excess temperatures in swirled jets of a viscous liquid and a gas, i.e. at $Pr_t \approx 1$, has been obtained. The results of experimental investigations of the fields of velocities, pressure and gas admixture concentration in axisymmetric turbulent swirled jets are presented which are compared with the solution obtained.

LAMINAR SWIRLED JET

The equation of heat transfer in a laminar boundary layer of a viscous incompressible liquid in the case of axisymmetric motion in a cylindrical coordinate system is

$$u\frac{\partial \Delta T}{\partial x} + v\frac{\partial \Delta T}{\partial r} = a\left(\frac{\partial^2 \Delta T}{\partial r^2} + \frac{1}{r}\frac{\partial \Delta T}{\partial r}\right). \tag{1}$$

Besides the momentum, K_0, and the moment of momentum, L_0, conservation conditions [5], the integral invariant of the problem is also the condition of the excess heat content conservation

$$Q_0 = 2\pi\rho c_p \int_0^\infty u\Delta T r \, dr = \text{const.} \tag{2}$$

Introducing new independent variables [5]

$$X = x, \quad \eta = r(x\nu)^{-1}, \tag{3}$$

we add the temperature expansion

$$\Delta T = \sum_{n=1}^{\infty} d_n(\eta)x^{-n}, \tag{4}$$

to the asymptotic expansions of the longitudinal, u, and transverse, v, velocity components, swirling rate, w, and pressure, P, of the 'dynamic' problem [3, 5, 6].

Substitution of equation (4) into equation (1) and comparison of the coefficients at the terms involving the same exponents of x yield a system of ordinary differential equations to determine the unknown functions, $d_1, d_2, d_3, \ldots$, which in terms of the variable

$$\zeta = \frac{\frac{1}{4}\alpha^2\eta^2}{1+\frac{1}{4}\alpha^2\eta^2}, \tag{5}$$

have the form [9]

$$\zeta(1-\zeta)d_1'' + [1-2(1-Pr)\zeta]d_1' + 2Pr\, d_1 = 0,$$

$$\zeta(1-\zeta)d_2'' + [1-2(1-Pr)\zeta]d_2' + 4Pr\, d_2 = \frac{\beta\bar{\alpha}^2}{2} Pr(1-4\zeta)(1-\zeta)^{2Pr},$$

$$\zeta(1-\zeta)d_3'' + [1-2(1-Pr)\zeta]d_3' + 6Pr\, d_3 = -\frac{\beta^2\bar{\alpha}^2}{2} Pr[\tfrac{3}{2} - (3Pr+9)\zeta + (12Pr+6)\zeta^2] \times (1-\zeta)^{2Pr} - \frac{\gamma^2\bar{\alpha}^2}{3\alpha^4} Pr[5 + (10Pr-19)\zeta - 14(Pr-1)\zeta^2](1-\zeta)^{2Pr-1}. \tag{6}$$

The boundary conditions are: $d_k(0)$ (limited) and $d_k(\infty) = 0$ $(k = 1, 2, \ldots)$ [9].

The solution of the first equation of system (6), which satisfies the boundary and integral (2) conditions [10] is

$$d_1(\zeta) = 2\bar{\alpha}^2(1-\zeta)^{2Pr}. \tag{7}$$

Here

$$\bar{\alpha} = \left[\frac{Q_0(1+2Pr)}{16\pi\mu c_p}\right]^{1/2}.$$

The homogeneous equations corresponding to d_2 and d_3, system (6), are hypergeometrical

$$\zeta(1-\zeta)d_k'' + [1-2(1-Pr)\zeta]d_k' + 2k\,Pr\, d_k = 0, \quad k = 2, 3,$$

and have the solution [11]

$$d_k = c_1F(a_k, b_k, 1, \zeta) + c_2\Big\{F(a_k, b_k, 1, \zeta)\ln\zeta + \sum_{n=1}^{\infty}\zeta^n\frac{(a_k)_n(b_k)_n}{(n!)^2}[\psi(a_k+n) - \psi(a_k) + \psi(b_k+n) - 2\psi(n+1) + 2\psi(n)]\Big\}, \quad k = 2, 3. \tag{8}$$

Here

$$a_k = \tfrac{1}{2} - Pr + \tfrac{1}{2}[1+4(2k-1)Pr + 4Pr^2]^{1/2},$$

$$b_k = \tfrac{1}{2} - Pr - \tfrac{1}{2}[1+4(2k-1)Pr + 4Pr^2]^{1/2}.$$

The quantity ψ with a respective argument is the Euler psi-function, and c_1 and c_2 are the integration constants. A partial integral of the inhomogeneous equation for d_2 of system (6) [9] is

$$d_2(\zeta) = -\frac{\beta^2\bar{\alpha}^2}{2}(1-4Pr\,\zeta)(1-\zeta)^{2Pr}. \tag{9}$$

According to the boundary conditions, $c_1 = c_2 = 0$ and the solution of the equation is only its partial integral, equation (9).

The solution of the equation for d_3 at $Pr = 1$ has been obtained elsewhere [8]. The general solution of the equation for d_3 at $Pr \neq 1$, which satisfies the boundary and integral conditions, can be found in a similar way as in ref. [8] and is of the form

$$d_3(\zeta) = \Big\{\frac{\beta^2\bar{\alpha}^2}{8}[1 - 10Pr\,\zeta + 4Pr(2Pr+1)\zeta^2] - \frac{\gamma^2\bar{\alpha}^2}{3\alpha^4(Pr-1)}[1 + Pr(5Pr-7)\zeta + Pr(Pr-1)] \times \sum_{n=2}\frac{a_n}{n!}\zeta^n\Big\}(1-\zeta)^{2Pr},$$

$$a_2 = 0, \quad \frac{a_n}{n!} = -\frac{4Pr}{n^2} + \frac{a_{n-1}}{(n-1)!} \times \left(\frac{n-1}{n} + 2Pr\frac{n-3}{n^2}\right), \quad n \geq 3. \tag{10}$$

Eventually, the temperature distribution in a swirled jet in terms of the variable η, equations (3), at $Pr \neq 1$ is

$$\Delta T = 2\bar{\alpha}^2B^{2Pr}\Big\{\frac{1}{x} - \frac{\beta}{4}\left[1 + (1-4Pr)\frac{\alpha^2\eta^2}{4}\right]B\frac{1}{x^2} + \left[\frac{\beta^2}{16}B^2\left[1 + \tfrac{1}{4}\alpha^2\eta^2(2-10Pr) + \frac{\alpha^4\eta^4}{16}(1-6Pr+8Pr^2)\right] - \frac{\gamma^2}{3(Pr-1)\alpha^4} \times \left(B[1+\tfrac{1}{4}\alpha^2\eta^2(1-7Pr+5Pr^2)] + Pr(Pr-1)\sum_{n=2}^{\infty}\frac{a_n}{n!}[\tfrac{1}{4}\alpha^2\eta^2B]^n\right)\right]\frac{1}{x^3}\Big\}. \tag{11}$$

Here

$$B = (1+\tfrac{1}{4}\alpha^2\eta^2)^{-1}.$$

TURBULENT SWIRLED JETS

Following the Loitsyansky hypothesis [5], an axisymmetric turbulent swirled jet will be considered as a laminar one, but as having molar viscosity. Consequently, the results obtained for the laminar jet are assumed to be valid for a turbulent one provided that the velocities are averaged in time and that the coefficients of molecular, μ, and kinematic, v, viscosity are substituted by the coefficients of molar viscosity A and kinematic eddy viscosity $v_t = A/\rho$, the values of which are determined according to ref. [12]. In conformity with this, the laminar flow Prandtl number

Table 1. Integral and characteristic constants of swirled jets

Type of jet	Ω	K_0 (N)	$L_0 \times 10^2$ (N m)	$\alpha/d^{1/2}$	β/d	γ/d^2	δ/d^3	$\bar{\alpha}$	k_η
'S-1'	1.86	1.16	0.553	11.72	10.20	54.77	559	0.140	0.67
'S-2'	2.08	1.06	0.552	11.62	8.89	235.26	575	0.132	0.76
'S-3'	2.48	1.30	0.804	11.40	6.16	609.92	1059	0.116	0.75

is substituted by the turbulent Prandtl number, Pr_t. Since the process of mass transport in jets is equivalent to the process of heat transfer [1, 10], these processes are described by the same equations with respective replacement of temperature, ΔT, by the expression for the concentration, c, and of the thermal diffusivity coefficient by the coefficient of diffusion, i.e. Pr_t by Sc. Hence, the distribution of admixture concentration in turbulent submerged jets is described by equation (14) taking account of the above remarks. According to ref. [1], $Sc = 0.7$. The velocity and pressure distributions in the jet are given elsewhere [3, 5, 6].

Experimental investigation of the gas admixture concentration distribution has been carried out for an axisymmetric turbulent swirled air jet escaping from a 10.5 mm diameter nozzle (the gas flow rate is 24 $m^3 h^{-1}$, $Re = 5.6 \times 10^4$). A gas admixture (methane) was introduced to the extent of 1% of the airflow rate into the pipeline which fed air to the nozzle. Swirled jets were formed with the aid of four-start screw swirlers sequentially inserted in the body of the nozzle. A gas–air jet was developed in a cylindrical tube of 800 mm in diameter which was connected to a smoke exhauster. The ratio of the airflow rates through the nozzle and the tube was 1 : 10. The velocity and the pressure in the jets were measured with a spherical five-channel probe (with the head having a diameter of 4 mm). This made it possible to obtain three components of the velocity and static pressure distributions. The Pitot–Prandtl tube was used as a gas sampling pipe. The secondary measuring devices were the MMN-250 micromanometers and gas chromatograph 'Tsvet' (Colour) with a recording potentiometer KSP-4. The measurement of methane concentration in the mixture was accurate to within 3%.

A criterion for the estimation of the swirling intensity was chosen to be the quantity $\Omega = 4L_0/K_0 d$.

The integral characteristics of the jets investigated are presented in Table 1. The characteristic constants α, β, γ, δ, and $\bar{\alpha}$ are determined by comparing the axial velocity, pressure and concentration distributions obtained experimentally with those predicted at $\alpha\eta = 0$ [3].

The variable η is connected with the jet radius by the formula $\alpha\eta = k_\eta r/x$ [3], where k_η is the proportionality factor the values of which are listed in Table 1.

Figures 1 and 2 present the experimental velocity and pressure distributions in swirled jets [3]. Here, the solid curves show the solution of the problem by the method of asymptotic expansions within the framework of the Loitsyansky theory [5] and of its modifications [3, 6]. Leaving aside the solution of the dynamic problem, the analysis of which is given elsewhere [3], we shall mention some specific features of the swirled jet development.

Figure 1(a) shows a change in the pressure, $P^* = 2P_{axis}/\rho U^2$, in maximum longitudinal velocities, $\bar{u}_m = u_m/U$, and in longitudinal velocities at the axis, $u^* = u_{axis}/U$, along the jets studied. It is seen that in the jets which are characterized by the presence of 'dips' in the longitudinal velocities at the jet axes, the maximum values of u^* do not correspond to the absence of 'dips' in the cross-section. The value of the longitudinal velocity on the jet axis becomes a maximum, $u^* = \bar{u}_m$, for all of the jets studied in the cross-sections spaced at $\bar{x} > 10$ from the nozzle edge. Figures 1(b) and (c) show variations of the longitudinal, $\bar{u} = u/U$, and radial, $\bar{v} = v/U$, components of the velocity vector over the jet cross-sections. It is seen from these figures that the appearance of the zone of 'dips' in the longitudinal velocities in swirled jets is accompanied by the appearance of negative radial velocities near the jet axis. A rapid increase of the longitudinal velocity on the jet axis with distance from the nozzle edge corresponds to an increase in the radial velocities near the jet axis and to their circumferential decrease. At the cross-section, $\bar{x} > 10$, the absence of 'dips' in the longitudinal velocities, of positive radial velocities near the jet axis and negative on the periphery are observed. Figures 2(a) and (b) present the variation in the swirling rate, $\bar{w} = w/U$, and in pressure, $\bar{P} = 2P/\rho U^2$, in swirled jets. Here the solid lines correspond to the solution of the problem by the method of asymptotic expansions [3].

Figure 3 shows the variation of the maximum concentrations, $\bar{c}_m = c_m/c_0$, along the axes of the jets studied, $\bar{x} = x/d$. The dashed curves correspond to the self-similar solution [the concentration expansion, formula (11), involves the first term at $\alpha\eta = 0$]; the solid curves are obtained by formula (11). It is seen from Fig. 3 that the boundary between the translational and the main (similarity) segments of the jets can be considered to lie at the cross-section where curves 1 and 2 practically converge. This distance from the nozzle edge depends on the swirling intensity, Ω, and for the jets studied it is equal to: 'S-1' – $\bar{x} = 25$, 'S-2' – $\bar{x} = 30$, 'S-3' – $\bar{x} = 34$. Since with an increasing Ω, the boundary of the similarity zone moves away from the nozzle edge and this leads to a change in the transition zone boundary, then for 'S-1' the solution obtained allows a description of the concentration distribution in the transition zone starting from the tenth diameter (the difference between the maximum predicted and

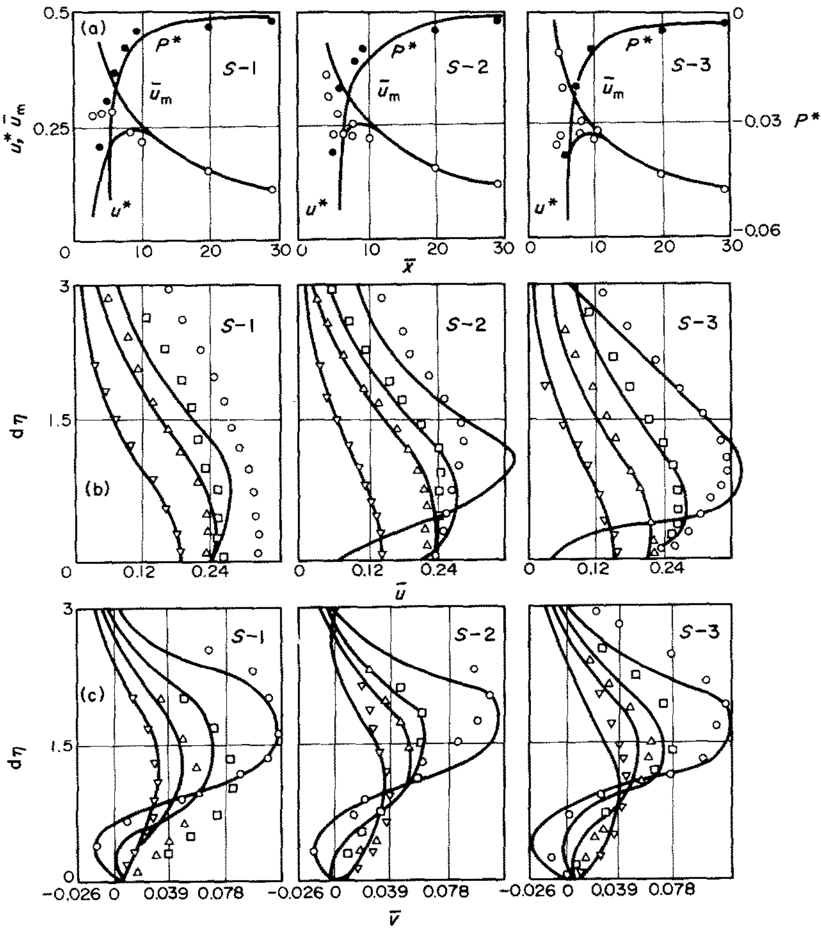

FIG. 1. Variation of pressure P^*, maximum longitudinal $\bar{u}_m$ and longitudinal axial u^* velocities along the axes of (a) swirled jets, (b) longitudinal $\bar{u}$ and (c) radial $\bar{v}$ velocities over the cross-sections of jets. ○, $x/d = 6$; □, $x/d = 8$; △, $x/d = 10$; ▽, $x/d = 20$.

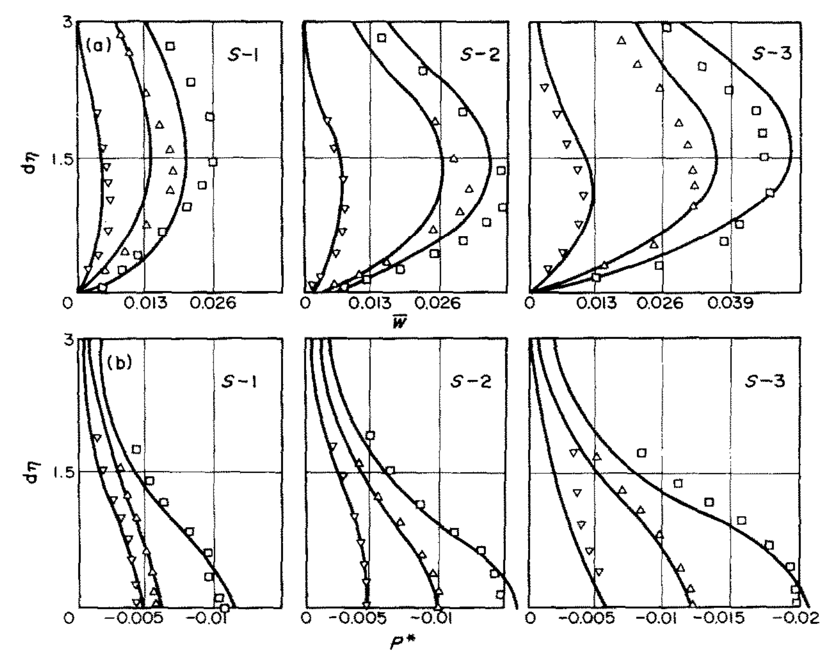

FIG. 2. Variation of (a) swirling rate $\bar{w}$ and (b) pressure $\bar{P}$ in swirled jets. □, $x/d = 8$; △, $x/d = 10$; ▽, $x/d = 20$.

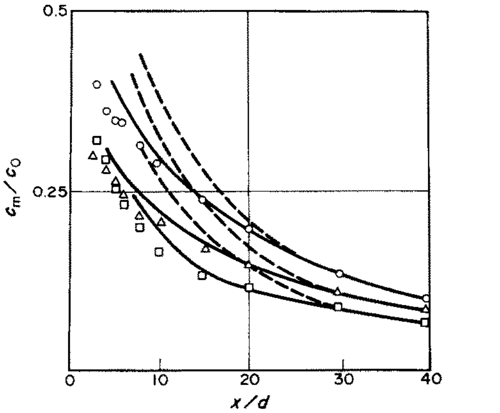

FIG. 3. Variation of maximum concentration of gas admixture $\bar{c}_m$ along the axes $\bar{x}$ of swirled jets: ○, 'S-1'; △, 'S-2'; □, 'S-3'; ---, self-similar solution; ——, non-self-similar solution.

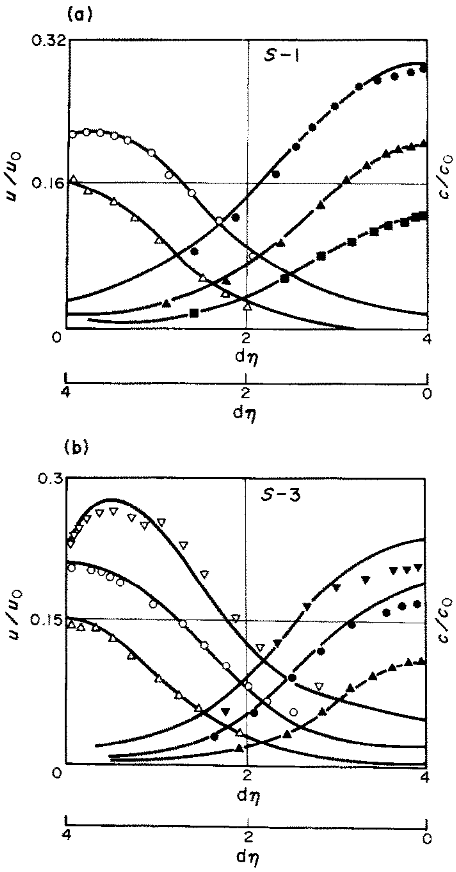

FIG. 4. Distribution of velocity $\bar{u}$ and concentration $\bar{c}$ over the cross-sections of swirled jets: 1, $x/d = 8$; 2, 10; 3, 20; 4, 30; light symbols, velocities (▽, 1; ○, 2; △, 3); dark symbols, concentrations (▼, 1; ●, 2; ▲, 3; ■, 4).

experimental concentration values is 3%); for 'S-2'—starting from the fifteenth diameter (the difference amounts to 5%) and for 'S-3'—starting from the twentieth diameter.

Figures 4(a) and (b) present the distributions of velocities, $\bar{u} = u/U$, calculated by the formulae of refs. [5, 6] [see Fig. 1(a)] and of concentration, $\bar{c} = c/c_0$, calculated by formula (11), for the jets 'S-1', 'S-3' over the cross-sections $\alpha\eta$ of the transitional and main portions of the jets. It is seen from these figures that the region of the jet characterized by a 'dip' in the axial velocity is well described by the formulae for the velocity, but as regards the concentration, there is some difference between the predicted and experimental results.

REFERENCES

1. G. N. Abramovich, S. Yu. Krasheninnikov, A. N. Sekundov and I. P. Smirnova, *Turbulent Mixing of Gas Jets*. Izd. Nauka, Moscow (1974).
2. R. B. Akhmedov, *Blast Gas-burners*. Izd. Nedra, Moscow (1970).
3. V. I. Korobko, *The Theory of Non-self-similar Viscous Liquid Jets*. Izd. Saratovsk. Gosuniver., Saratov (1977).
4. M. A. Goldshtik, *Eddy Fluxes*. Izd. Nauka, Novosibirsk (1981).
5. L. G. Loitsyansky, The propagation of a swirled jet in an infinite space submerged in the same liquid, *Prikl. Mat. Mekh.* **17**(1), 3–16 (1953).
6. S. V. Falkovich, The propagation of a swirled jet in an infinite space submerged in the same liquid, *Prikl. Mat. Mekh.* **32**(1), 282–288 (1967).
7. V. I. Korobko and S. V. Falkovich, The development of a swirled jet in an infinite space, *Izv. Akad. Nauk SSSR, Mekh. Zhid. Gaza* No. 3, 56–63 (1969).
8. S. V. Falkovich and V. I. Korobko, The aerodynamics and heat transfer of a swirled jet propagating in an infinite space submerged in the same liquid, *Izv. VUZov, Mat.* No. 7, 87–95 (1969).
9. Z. P. Shulman, V. I. Korobko and V. K. Shashmin, Heat and mass transfer in a submerged axisymmetric non-self-similar jet, *J. Engng Phys.* **41**(4), 645–650 (1981).
10. L. A. Vulis and V. P. Kashkarov, *The Theory of Viscous Liquid Jets*. Izd. Nauka, Moscow (1965).
11. G. Beitmen and A. Erdeii, *Higher Transcendental Functions. Hypergeometrical Functions. Legendre Functions*. Izd. Nauka, Moscow (1965).
12. L. G. Loitsyansky, *Liquid and Gas Mechanics*. Izd. Nauka, Moscow (1970).

TRANSFERT MASSIQUE ET THERMIQUE DANS DES JETS TOURBILLONNAIRES VISQUEUX

Résumé—Dans le cadre de la théorie de Loitsyansky pour le problème du développement des jets tourbillonnaires submergés, la distribution des températures en excès a été obtenue par la méthode des développements asymptotiques. Le second et le troisième termes du développement de la température ont été obtenus en tenant compte de l'effet de l'intensité du tourbillonnement pour un large domaine de nombre de Prandtl ($Pr \neq 1$). Les résultats des recherches expérimentales sur les champs de vitesse, de pression et de concentration d'un gaz dans des jets tourbillonnaires et turbulents d'air sont donnés et ils sont comparés avec le calcul.

WÄRME- UND STOFFÜBERTRAGUNG IN ZÄHEN WIRBELSTRAHLEN

Zusammenfassung—Im Rahmen der Loitsyanski-Theorie zum Problem der Ausbildung überfluteter Wirbelstrahlen wurde die Verteilung der Übertemperaturen in Wirbelstrahlen mit dem Verfahren der asymptotischen Reihenentwicklung berechnet. Die zweiten und dritten Terme der Entwicklung für die Übertemperatur wurden ermittelt, wobei der Einfluß der Wirbelintensität des Strahls über einen weiten Bereich der Prandtl-Zahl ($Pr \neq 1$) berücksichtigt wurde. Experimentelle Ergebnisse für die Geschwindigkeits-, Druck- und Konzentrationsfelder turbulenter Luft-Wirbelstrahlen bei Gas-Beimischung werden mitgeteilt und mit den Berechnungen verglichen.

ТЕПЛО- И МАССООБМЕН В ВЯЗКИХ ЗАКРУЧЕННЫХ СТРУЯХ

Аннотация—Методом асимптотических разложений в рамках теории Лойцянского задачи о развитиии затопленных закрученных струй получено распределение избыточных температур в закрученных струях. Найдены второй и третий члены разложения избыточных температур, что позволяет учесть влияние интенсивности закрутки струи для широкого диапазона чисел $Pr \neq 1$. Приведены результаты экспериментальных исследований полей скоростей, давления и концентрации газовой примеси в турбулентных закрученных струях воздуха, которые сопоставлены с решением.